Chapter 1

Prerequisites for Calculus

Section 1.1 Lines (pp. 3–11)

Quick Review 1.1

1. $y = -2 + 4(3 - 3) = -2 + 4(0) = -2 + 0 = -2$

2. $$\begin{aligned} 3 &= 3 - 2(x+1) \\ 3 &= 3 - 2x - 2 \\ 2x &= -2 \\ x &= -1 \end{aligned}$$

3. $m = \dfrac{2-3}{5-4} = \dfrac{-1}{1} = -1$

4. $m = \dfrac{2-(-3)}{3-(-1)} = \dfrac{5}{4}$

5. **(a)** $3(2) - 4\left(\dfrac{1}{4}\right) \stackrel{?}{=} 5$
$6 - 1 = 5$ Yes

(b) $3(3) - 4(-1) \stackrel{?}{=} 5$
$13 \neq 5$ No

6. **(a)** $7 \stackrel{?}{=} -2(-1) + 5$
$7 = 2 + 5$ Yes

(b) $1 = -2(-2) + 5$
$1 \neq 9$ No

7. $$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(0-1)^2 + (1-0)^2} \\ &= \sqrt{2} \end{aligned}$$

8. $$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1-2)^2 + \left(-\frac{1}{3} - 1\right)^2} \\ &= \sqrt{(-1)^2 + \left(-\frac{4}{3}\right)^2} \\ &= \sqrt{1 + \frac{16}{9}} \\ &= \sqrt{\frac{25}{9}} \\ &= \frac{5}{3} \end{aligned}$$

9. $$\begin{aligned} 4x - 3y &= 7 \\ -3y &= -4x + 7 \\ y &= \frac{4}{3}x - \frac{7}{3} \end{aligned}$$

10. $$\begin{aligned} -2x + 5y &= -3 \\ 5y &= 2x - 3 \\ y &= \frac{2}{5}x - \frac{3}{5} \end{aligned}$$

Section 1.1 Exercises

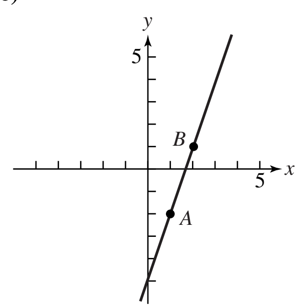
1. $\Delta x = -1 - 1 = -2$
$\Delta y = -1 - 2 = -3$

2. $\Delta x = -1 - (-3) = 2$
$\Delta y = -2 - 2 = -4$

3. $\Delta x = -8 - (-3) = -5$
$\Delta y = 1 - 1 = 0$

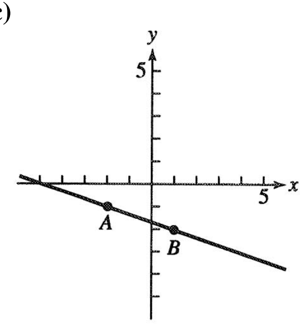
4. $\Delta x = 0 - 0 = 0$
$\Delta y = -2 - 4 = -6$

5. **(a, c)**



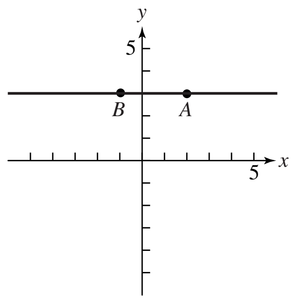
(b) $m = \dfrac{1-(-2)}{2-1} = \dfrac{3}{1} = 3$

6. **(a, c)**



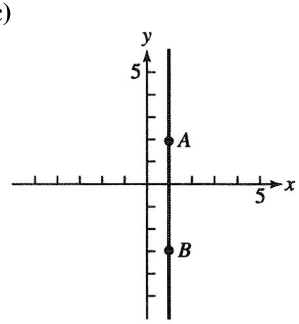
(b) $m = \dfrac{-2-(-1)}{1-(-2)} = \dfrac{-1}{3} = -\dfrac{1}{3}$

7. **(a, c)**



(b) $m = \frac{3-3}{-1-2} = \frac{0}{-3} = 0$

8. **(a, c)**



(b) $m = \frac{-3-2}{1-1} = \frac{-5}{0}$ (undefined)

This line has no slope.

9. **(a)** $x = 3$

(b) $y = 2$

10. **(a)** $x = -1$

(b) $y = \frac{4}{3}$

11. **(a)** $x = 0$

(b) $y = -\sqrt{2}$

12. **(a)** $x = -\pi$

(b) $y = 0$

13. $y = 1(x - 1) + 1$

14. $y = -1[x - (-1)] + 1$
$y = -1(x + 1) + 1$

15. $y = 2(x - 0) + 3$

16. $y = -2[x - (-4)] + 0$
$y = -2(x + 4) + 0$

17. $y = 3x - 2$

18. $y = -1x + 2$ or $y = -x + 2$

19. $y = -\frac{1}{2}x - 3$

20. $y = \frac{1}{3}x - 1$

21. $m = \frac{3-0}{2-0} = \frac{3}{2}$

$$y = \frac{3}{2}(x-0)+0$$
$$y = \frac{3}{2}x$$
$$2y = 3x$$
$$3x - 2y = 0$$

22. $m = \frac{1-1}{2-1} = \frac{0}{1} = 0$

$y = 0(x - 1) + 1$
$y = 1$

23. $m = \frac{-2-0}{-2-(-2)} = \frac{-2}{0}$ (undefined)

Vertical line: $x = -2$

24. $m = \frac{-2-1}{2-(-2)} = \frac{-3}{4} = -\frac{3}{4}$

$$y = -\frac{3}{4}[x-(-2)]+1$$
$$4y = -3(x+2)+4$$
$$4y = -3x-2$$
$$3x+4y = -2$$

25. The line contains (0, 0) and (10, 25).

$m = \frac{25-0}{10-0} = \frac{25}{10} = \frac{5}{2}$

$y = \frac{5}{2}x$

26. The line contains (0, 0) and (5, 2).

$m = \frac{2-0}{5-0} = \frac{2}{5}$

$y = \frac{2}{5}x$

27. $3x+4y=12$

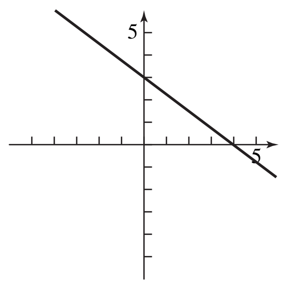
$$4y=-3x+12$$

$$y=-\frac{3}{4}x+3$$

(a) Slope: $-\frac{3}{4}$

(b) y-intercept: 3

(c)



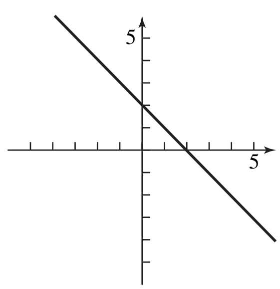
28. $x+y=2$

$$y=-x+2$$

(a) Slope: -1

(b) y-intercept: 2

(c)



29. $\frac{x}{3}+\frac{y}{4}=1$

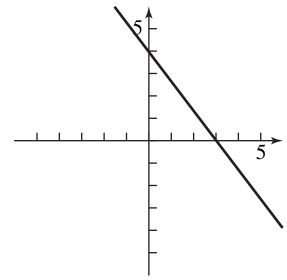
$$\frac{y}{4}=-\frac{x}{3}+1$$

$$y=-\frac{4}{3}x+4$$

(a) Slope: $-\frac{4}{3}$

(b) y-intercept: 4

(c)

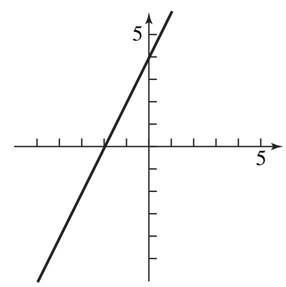


30. $y = 2x + 4$

(a) Slope: 2

(b) y-intercept: 4

(c)



31. The Line L: $y = -x + 2$ has slope -1.

(a) The desired line has slope -1 and passes through (0, 0): $y = -1(x - 0) + 0$ or $y = -x$.

(b) The desired line has slope $\frac{-1}{-1}=1$ and passes through (0, 0): $y = 1(x - 0) + 0$ or $y = x$.

32. The given equation is equivalent to $y = -2x + 4$, so L has slope -2.

(a) The desired line has slope -2 and passes through $(-2, 2)$: $y = -2(x + 2) + 2$ or $y = -2x - 2$.

(b) The desired line has slope $\frac{-1}{-2}=\frac{1}{2}$ and passes through $(-2, 2)$: $y=\frac{1}{2}(x+2)+2$ or $y=\frac{1}{2}x+3.$

33. The line L: $x = 5$ is vertical and has no slope.

(a) We seek a vertical line through $(-2, 4)$: $x = -2$.

(b) We seek a horizontal line through (–2, 4): $y = 4$.

34. The line L: $y = 3$ is horizontal and has slope 0.

(a) We seek a horizontal line through $\left(-1, \frac{1}{2}\right)$: $y = \frac{1}{2}$.

(b) We seek a vertical line through $\left(-1, \frac{1}{2}\right)$: $x = -1$.

35. $m = \frac{9-2}{3-1} = \frac{7}{2}$

$f(x) = \frac{7}{2}(x-1)+2 = \frac{7}{2}x - \frac{3}{2}$

Check: $f(5) = \frac{7}{2}(5) - \frac{3}{2} = 16$, as expected.

Since $f(x) = \frac{7}{2}x - \frac{3}{2}$, we have $m = \frac{7}{2}$ and $b = -\frac{3}{2}$.

36. $m = \frac{-4-(-1)}{4-2} = \frac{-3}{2} = -\frac{3}{2}$

$f(x) = -\frac{3}{2}(x-2)+(-1) = -\frac{3}{2}x+2$

Check: $f(6) = -\frac{3}{2}(6)+2 = -7$, as expected.

Since $f(x) = -\frac{3}{2}x+2$, we have $m = -\frac{3}{2}$ and $b = 2$.

37.
$$
\begin{aligned}
-\frac{2}{3} &= \frac{y-3}{4-(-2)} \\
-\frac{2}{3}(6) &= y-3 \\
-4 &= y-3 \\
-1 &= y
\end{aligned}
$$

38.
$$
\begin{aligned}
2 &= \frac{2-(-2)}{x-(-8)} \\
2(x+8) &= 4 \\
x+8 &= 2 \\
x &= -6
\end{aligned}
$$

39. $y = 1 \cdot (x-3)+4$
$y = x - 3 + 4$
$y = x + 1$
This is the same as the equation obtained in Example 4.

40. **(a)** When $y = 0$, we have $\frac{x}{c} = 1$, so $x = c$.

When $x = 0$, we have $\frac{y}{d} = 1$, so $y = d$.

(b) When $y = 0$, we have $\frac{x}{c} = 2$, so $x = 2c$.

When $x = 0$, we have $\frac{y}{d} = 2$, so $y = 2d$.

The x-intercept is $2c$ and the y-intercept is $2d$.

41. The given equations are equivalent to $y = -\frac{2}{k}x + \frac{3}{k}$ and $y = -x + 1$, respectively, so the slopes are $-\frac{2}{k}$ and -1.

(a) The lines are parallel when $-\frac{2}{k} = -1$, so $k = 2$.

(b) The lines are perpendicular when $-\frac{2}{k} = \frac{-1}{-1}$, so $k = -2$.

42. **(a)**
$$
\begin{aligned}
m &= \frac{68-69.5}{0.4-0} \\
&= \frac{-1.5}{0.4} \\
&= -3.75 \text{ degrees/inch}
\end{aligned}
$$

(b) $m = \frac{9-68}{4-0.4} = \frac{-59}{3.6} = -16.4$ degrees/inch

(c) $m = \frac{4-9}{4.7-4} = \frac{-5}{0.7} = -7.1$ degrees/inch

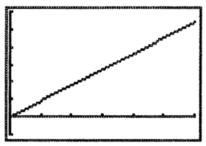
(d) Best insulator: Fiberglass insulation
Poorest insulator: Gypsum wallboard
The best insulator will have the largest temperature change per inch, because that will allow larger temperature differences on opposite sides of thinner layers.

43. Slope: $k = \frac{\Delta p}{\Delta d}$
$= \frac{10.94 - 1}{100 - 0}$
$= \frac{9.94}{100}$
$= 0.0994$ atmospheres per meter

At 50 meters, the pressure is
$p = 0.0994(50) + 1 = 5.97$ atmospheres.

44. **(a)** $d(t) = 45t$

(b)



[0, 6] by [−50, 300]

(c) The slope is 45, which is the speed in miles per hour.

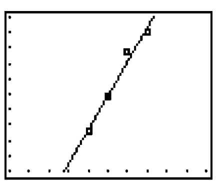
(d) Suppose the car has been traveling 45 mph for several hours when it is first observed at point P at time $t = 0$.

(e) The car starts at time $t = 0$ at a point 30 miles past P.

45. **(a)** $y = 2216.2x - 4{,}387{,}470.6$

(b) The slope is 2216.2. It represents the approximate rate of increase in earnings in dollars per year.

(c)



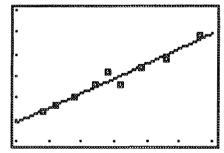
[1995, 2005] by [40000, 50000]

(d) When $x = 2008$,
$y = 2216.2(2008) - 4{,}387{,}470.6$
$= 62{,}659$.
In 2008, the construction workers' average annual compensation will be about $62,659.

46. **(a)** $y = 0.680x + 9.013$

(b) The slope is 0.68. It represents the approximate average weight gain in pounds per month.

(c)



[15, 45] by [15, 45]

(d) When $x = 30$,
$y = 0.680(30) + 9.013 = 29.413$.
She weighs about 29 pounds.

47. False: $m = \frac{\Delta y}{\Delta x}$ and $\Delta x = 0$, so it is undefined, or has no slope.

48. False: perpendicular lines satisfy the equation $m_1 m_2 = -1$, or $m_1 = -\frac{1}{m_2}$.

49. A: $y = m(x - x_1) + y_1$
$y = \frac{1}{2}(x+3) + 4$ or $y - 4 = \frac{1}{2}(x+3)$

50. E

51. D: $y = 2x - 5$
$0 = 2x - 5$
$5 = 2x$
$x = \frac{5}{2}$

52. B: $m = -3$
$y = -3(x - (-2)) + (-1)$
$y = -3(x + 2) - 1$
$y = -3x - 7$

53. **(a)** $y = 5980x - 11{,}809{,}820$

(b) The rate at which the median price is increasing in dollars per year

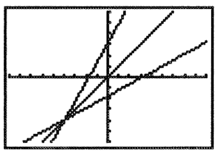
(c) $y = 21{,}650x - 43{,}105{,}030$

(d) The median price is increasing at a rate of about $5980 per year in the South, and about $21,650 per year in the West. It is increasing more rapidly in the West.

54. **(a)** Suppose x°F is the same as x°C.

$$x = \frac{9}{5}x + 32$$
$$x - \frac{9}{5}x = 32$$
$$\left(1 - \frac{9}{5}\right)x = 32$$
$$-\frac{4}{5}x = 32$$
$$x = -40$$

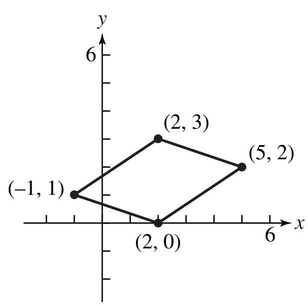
Yes, –40°F is the same as –40°C.

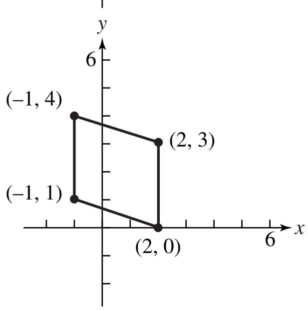
(b) 

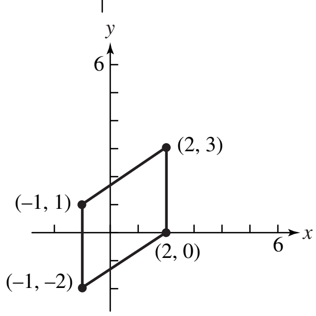
[−90, 90] by [−60, 60]

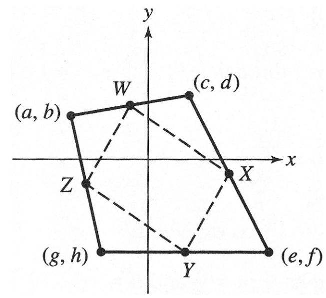
It is related because all three lines pass through the point (–40, –40), where the Fahrenheit and Celsius temperatures are the same.

55. The coordinates of the three missing vertices are (5, 2), (–1, 4) and (–1, –2), as shown below.







56. 

Suppose that the vertices of the given quadrilateral are (a, b), (c, d), $e, f)$, and (g, h). Then the midpoints of the consecutive sides are $W\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$, $X\left(\frac{c+e}{2}, \frac{d+f}{2}\right)$, $Y\left(\frac{e+g}{2}, \frac{f+h}{2}\right)$, and $Z\left(\frac{g+a}{2}, \frac{h+b}{2}\right)$. When these four points are connected, the slopes of the sides of the resulting figure are:

$$WX: \frac{\frac{d+f}{2} - \frac{b+d}{2}}{\frac{c+e}{2} - \frac{a+c}{2}} = \frac{f-b}{e-a}$$

$$XY: \frac{\frac{f+h}{2} - \frac{d+f}{2}}{\frac{e+g}{2} - \frac{c+e}{2}} = \frac{h-d}{g-c}$$

$$ZY: \frac{\frac{f+h}{2} - \frac{h+b}{2}}{\frac{e+g}{2} - \frac{g+a}{2}} = \frac{f-b}{e-a}$$

$$WZ: \frac{\frac{h+b}{2} - \frac{b+d}{2}}{\frac{g+a}{2} - \frac{a+c}{2}} = \frac{h-d}{g-c}$$

Opposite sides have the same slope and are parallel.

57. The radius through (3, 4) has slope $\frac{4-0}{3-0} = \frac{4}{3}$. The tangent line is perpendicular to the radius, so its slope is $\frac{-1}{\frac{4}{3}} = -\frac{3}{4}$. We seek the line of slope $-\frac{3}{4}$ that passes through (3, 4).

$$y = -\frac{3}{4}(x-3) + 4$$
$$y = -\frac{3}{4}x + \frac{9}{4} + 4$$
$$y = -\frac{3}{4}x + \frac{25}{4}$$

58. **(a)** The equation for line L can be written as $y = -\frac{A}{B}x + \frac{C}{B}$, so its slope is $-\frac{A}{B}$. The perpendicular line has slope $\frac{-1}{-\frac{A}{B}} = \frac{B}{A}$ and passes through (a, b), so its equation is $y = \frac{B}{A}(x-a)+b$.

(b) Substituting $\frac{B}{A}(x-a)+b$ for y in the equation for line L gives:

$$Ax + B\left[\frac{B}{A}(x-a)+b\right] = C$$

$$A^2x + B^2(x-a) + ABb = AC$$

$$(A^2+B^2)x = B^2a + AC - ABb$$

$$x = \frac{B^2a + AC - ABb}{A^2+B^2}$$

Substituting the expression for x in the equation for line L gives:

$$A\left(\frac{B^2a + AC - ABb}{A^2+B^2}\right) + By = C$$

$$By = \frac{-A(B^2a + AC - ABb)}{A^2+B^2} + \frac{C(A^2+B^2)}{A^2+B^2}$$

$$By = \frac{-AB^2a - A^2C + A^2Bb + A^2C + B^2C}{A^2+B^2}$$

$$By = \frac{A^2Bb + B^2C - AB^2a}{A^2+B^2}$$

$$y = \frac{A^2b + BC - ABa}{A^2+B^2}$$

The coordinates of Q are $\left(\frac{B^2a + AC - ABb}{A^2+B^2}, \frac{A^2b + BC - ABa}{A^2+B^2}\right)$.

(c) Distance $= \sqrt{(x-a)^2+(y-b)^2}$

$$= \sqrt{\left(\frac{B^2a+AC-ABb}{A^2+B^2}-a\right)^2+\left(\frac{A^2b+BC-ABa}{A^2+B^2}-b\right)^2}$$

$$= \sqrt{\left(\frac{B^2a+AC-ABb-a(A^2+B^2)}{A^2+B^2}\right)^2+\left(\frac{A^2b+BC-ABa-b(A^2+B^2)}{A^2+B^2}\right)^2}$$

$$= \sqrt{\left(\frac{AC-ABb-A^2a}{A^2+B^2}\right)^2+\left(\frac{BC-ABa-B^2b}{A^2+B^2}\right)^2}$$

$$= \sqrt{\left(\frac{A(C-Bb-Aa)}{A^2+B^2}\right)^2+\left(\frac{B(C-Aa-Bb)}{A^2+B^2}\right)^2}$$

$$= \sqrt{\frac{A^2(C-Aa-Bb)^2}{(A^2+B^2)^2}+\frac{B^2(C-Aa-Bb)^2}{(A^2+B^2)^2}}$$

$$= \sqrt{\frac{(A^2+B^2)(C-Aa-Bb)^2}{(A^2+B^2)^2}}$$

$$= \sqrt{\frac{(C-Aa-Bb)^2}{A^2+B^2}}$$

$$= \frac{|C-Aa-Bb|}{\sqrt{A^2+B^2}}$$

$$= \frac{|Aa+Bb-C|}{\sqrt{A^2+B}}$$

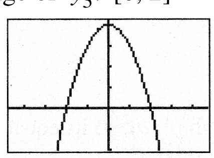
Section 1.2 Functions and Graphs
(pp. 12–21)

Exploration 1 Composing Functions

1. $y_3 = g \circ f,\ y_4 = f \circ g$

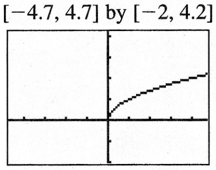
2. Domain of y_3: $[-2, 2]$
Range of y_3: $[0, 2]$

y_1:



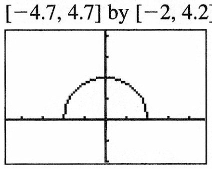
$[-4.7, 4.7]$ by $[-2, 4.2]$

y_2:



$[-4.7, 4.7]$ by $[-2, 4.2]$

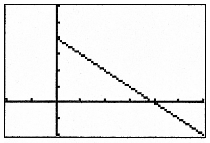
y_3:



$[-4.7, 4.7]$ by $[-2, 4.2]$

3. Domain of y_4: $[0, \infty)$

Range of y_4: $(-\infty, 4]$



$[-2, 6]$ by $[-2, 6]$

4. $y_3 = y_2(y_1(x)) = \sqrt{y_1(x)} = \sqrt{4-x^2}$

$y_4 = y_1(y_2(x))$
$= 4-(y_2(x))^2$
$= 4-(\sqrt{x})^2$
$= 4-x,\ x \ge 0$

Quick Review 1.2

1. $3x-1 \le 5x+3$
$-2x \le 4$
$x \ge -2$
Solution: $[-2, \infty)$

2. $x(x-2) > 0$
Solutions to $x(x-2)=0$: $x = 0$, $x = 2$
Test $x = -1$: $-1(-1-2) = 3 > 0$
$x(x-2) > 0$ is true when $x < 0$.
Test $x = 1$: $1(1-2) = -1 < 0$
$x(x-2) > 0$ is false when $0 < x < 2$.
Test $x = 3$: $3(3-2) = 3 > 0$
$x(x-2) > 0$ is true when $x > 2$.
Solution set: $(-\infty, 0) \cup (2, \infty)$

3. $|x-3| < 4$
$-4 \le x-3 \le 4$
$-1 \le x \le 7$
Solution set: $[-1, 7]$

4. $|x-2| \ge 5$
$x-2 \le -5$ or $x-2 \ge 5$
$x \le -3$ or $x \ge 7$
Solution set: $(-\infty, -3] \cup [7, \infty)$

5. $x^2 < 16$
Solutions to $x^2 = 16$: $x = -4$, $x = 4$
Test $x = -6$: $(-6)^2 = 36 > 16$
$x^2 < 16$ is false when $x < -4$.
Test $x = 0$: $0^2 = 0 < 16$
$x^2 < 16$ is true when $-4 < x < 4$.
Test $x = 6$: $6^2 = 36 > 16$
$x^2 < 16$ is false when $x > 4$.
Solution set: $(-4, 4)$

6. $9 - x^2 \ge 0$
Solutions to $9-x^2 = 0$: $x = -3$, $x = 3$
Test $x = -4$: $9-(-4)^2 = 9-16 = -7 < 0$
$9-x^2 \ge 0$ is false when $x < -3$.
Test $x = 0$: $9-0^2 = 9 > 0$
$9-x^2 \ge 0$ is true when $-3 < x < 3$.
Test $x = 4$: $9-4^2 = 9-16 = 7 < 0$
$9-x^2 \ge 0$ is false when $x > 3$.
Solution set: $[-3, 3]$

7. Translate the graph of f 2 units left and 3 units downward.

8. Translate the graph of f 5 units right and 2 units upward.

9. **(a)** $f(x) = 4$
$x^2 - 5 = 4$
$x^2 - 9 = 0$
$(x+3)(x-3) = 0$
$x = -3$ or $x = 3$

(b) $f(x) = -6$
$x^2 - 5 = -6$
$x^2 = -1$
No real solution

10. **(a)** $f(x) = -5$
$\frac{1}{x} = -5$
$x = -\frac{1}{5}$

(b) $f(x) = 0$
$\frac{1}{x} = 0$
No solution

11. **(a)** $f(x) = 4$
$\sqrt{x+7} = 4$
$x+7 = 16$
$x = 9$
Check: $\sqrt{9+7} = \sqrt{16} = 4$; it checks.

(b) $$\begin{aligned} f(x) &= 1 \\ \sqrt{x+7} &= 1 \\ x+7 &= 1 \\ x &= -6 \end{aligned}$$
Check: $\sqrt{-6+7} = 1$; it checks.

12. (a) $$\begin{aligned} f(x) &= -2 \\ \sqrt[3]{x-1} &= -2 \\ x-1 &= -8 \\ x &= -7 \end{aligned}$$

(b) $$\begin{aligned} f(x) &= 3 \\ \sqrt[3]{x-1} &= 3 \\ x-1 &= 27 \\ x &= 28 \end{aligned}$$

Section 1.2 Exercises

1. $A(d) = \pi\left(\dfrac{d}{2}\right)^2$

$A(4) = \pi\left(\dfrac{4 \text{ in.}}{2}\right)^2 = \pi(2 \text{ in.})^2 = 4\pi \text{ in}^2$

2. $h(s) = \dfrac{\sqrt{3}}{2}s$

$h(3) = \dfrac{\sqrt{3}}{2} \cdot 3 \text{ m} = 1.5\sqrt{3} \text{ m}$

3. $S(e) = 6e^2$

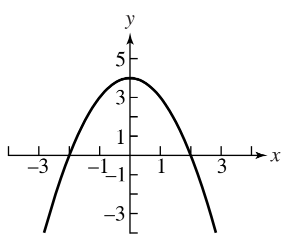
$S(5) = 6(5 \text{ ft})^2 = 6(25 \text{ ft}^2) = 150 \text{ ft}^2$

4. $V(r) = \dfrac{4}{3}\pi r^3$

$V(3) = \dfrac{4}{3}\pi(3 \text{ cm})^3 = \dfrac{4}{3}\pi(27 \text{ cm}^3) = 36\pi \text{ cm}^3$

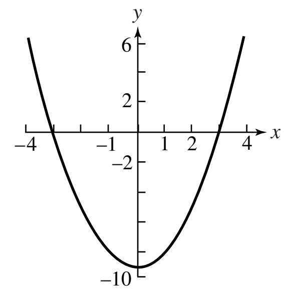
5. (a) Domain: $(-\infty, \infty)$ or all real numbers
Range: $(-\infty, 4]$

(b)



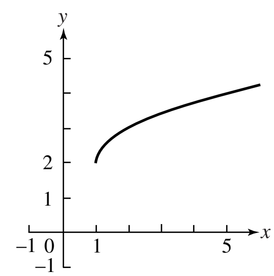
6. (a) Domain: $(-\infty, \infty)$ or all real numbers
Range: $[-9, \infty)$

(b)



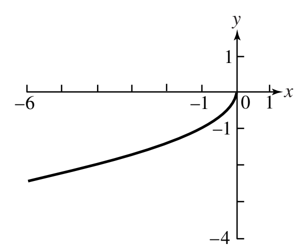
7. (a) Since we require $x - 1 \geq 0$, the domain is $[1, \infty)$. Range: $[2, \infty)$

(b)



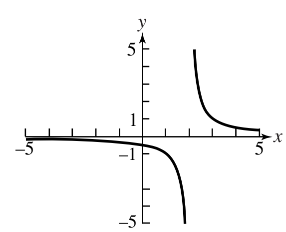
8. (a) Since we require $-x \geq 0$, the domain is $(-\infty, 0]$. Range: $(-\infty, 0]$

(b)



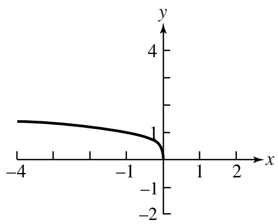
9. (a) Since we require $x - 2 \neq 0$, the domain is $(-\infty, 2) \cup (2, \infty)$. Since $\dfrac{1}{x-2}$ can assume any value except 0, the range is $(-\infty, 0) \cup (0, \infty)$.

(b)



10. **(a)** Since we require $-x \geq 0$, the domain is $(-\infty, 0]$. Range: $[0, \infty)$

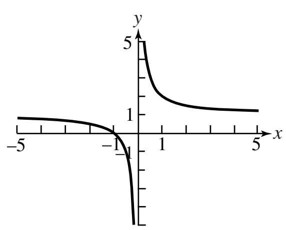
(b)



11. **(a)** Since we require $x \neq 0$, the domain is $(-\infty, 0) \cup (0, \infty)$.

Note that $\frac{1}{x}$ can assume any value except 0, so $1+\frac{1}{x}$ can assume any value except 1. The range is $(-\infty, 1) \cup (1, \infty)$.

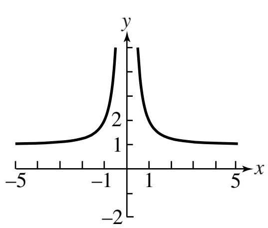
(b)



12. **(a)** Since we require $x^2 \neq 0$, the domain is $(-\infty, 0) \cup (0, \infty)$.

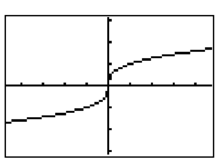
Since $\frac{1}{x^2} > 0$ for all x, the range is $(1, \infty)$.

(b)



13. **(a)** Domain: $(-\infty, \infty)$ or all real numbers
Range: $(-\infty, \infty)$ or all real numbers

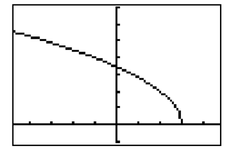
(b)



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

14. **(a)** Since we require $3 - x \geq 0$, the domain is $(-\infty, 3]$. Range: $[0, \infty)$

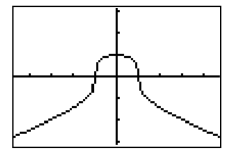
(b)



$[-4.7, 4.7]$ by $[-1, 7]$

15. **(a)** Domain: $(-\infty, \infty)$ or all real numbers
The maximum function value is attained at the point $(0, 1)$, so the range is $(-\infty, 1]$.

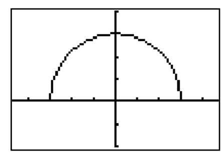
(b)



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

16. **(a)** Since we require $9 - x^2 \geq 0$ the domain is $[-3, 3]$. Range: $[0, 3]$

(b)

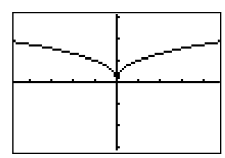


$[-4.7, 4.7]$ by $[-2, 4]$

17. **(a)** Domain: $(-\infty, \infty)$ or all real numbers

Since $x^{2/5}$ is equivalent to $\left(\sqrt[5]{x}\right)^2$ the range is $[0, \infty)$.

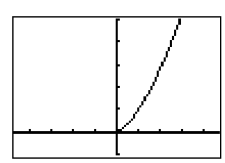
(b)



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

18. **(a)** This function is equivalent to $y = \left(\sqrt{x}\right)^3$, so its domain is $[0, \infty)$.
Range: $[0, \infty)$

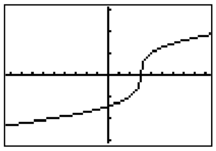
(b)



$[-4.7, 4.7]$ by $[-1, 5]$

19. **(a)** Domain: $(-\infty, \infty)$ or all real numbers
Range: $(-\infty, \infty)$ or all real numbers

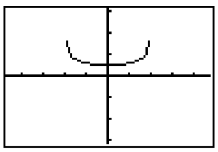
(b)



$[-9.4, 9.4]$ by $[-3.1, 3.1]$

20. **(a)** Since we require $4 - x^2 > 0$, the domain is $(-2, 2)$. Domain: $[0.5, \infty)$

(b)



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

(Note: this is an example of grapher failure because the graph really has vertical asymptotes at $x = \pm 2$ that do not show up in the graph above.)

21. Even, since the function is an even power of x.

22. Neither, since the function is a sum of even and odd powers of x.

23. Neither, since the function is a sum of even and odd powers of x, $(x^1 + 2x^0)$.

24. Even, since the function is a sum of even powers of x, $(x^2 - 3x^0)$.

25. Even, since the function involves only even powers of x.

26. Odd, since the function is a sum of odd powers of x.

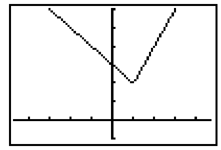
27. Odd, since the function is a quotient of an odd function (x^3) and an even function $(x^2 - 1)$.

28. Neither, since, (for example), $y(-2) = 4^{1/3}$ and $y(2) = 0$.

29. Neither, since, (for example), $y(-1)$ is defined and $y(1)$ is undefined.

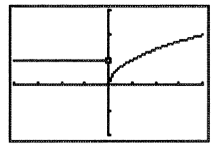
30. Even, since the function involves only even powers of x.

31.



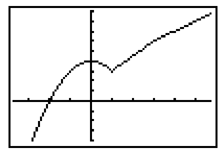
$[-4.7, 4.7]$ by $[-1, 6]$

32.



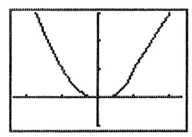
$[-4, 4]$ by $[-2, 3]$

33.



$[-3.7, 5.7]$ by $[-4, 9]$

34.



$[-2.35, 2.35]$ by $[-1, 3]$

35. Because if the vertical line test holds, then for each x-coordinate, there is at most one y-coordinate giving a point on the curve. This y-coordinate would correspond to the value assigned to the x-coordinate. Since there is only one y-coordinate, the assignment would be unique.

36. If the curve is not $y = 0$, there must be a point (x, y) on the curve where $y \neq 0$. That would mean that (x, y) and $(x, -y)$ are two different points on the curve and it is not the graph of a function, since it fails the vertical line test.

37. No

38. Yes

39. Yes

40. No

41. Line through (0, 0) and (1, 1): $y = x$
Line through (1, 1) and (2, 0): $y = -x + 2$

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ -x + 2, & 1 < x \le 2 \end{cases}$$

42. $f(x)=\begin{cases}2, & 0\le x<1\\ 0, & 1\le x<2\\ 2, & 2\le x<3\\ 0, & 3\le x\le 4\end{cases}$

43. Line through (0, 2) and (2, 0): $y = -x + 2$
Line through (2, 1) and (5, 0):
$m=\frac{0-1}{5-2}=\frac{-1}{3}=-\frac{1}{3}$ so
$y=-\frac{1}{3}(x-2)+1=-\frac{1}{3}x+\frac{5}{3}.$
$f(x)=\begin{cases}-x+2, & 0<x\le 2\\ -\frac{1}{3}x+\frac{5}{3}, & 2<x\le 5\end{cases}$

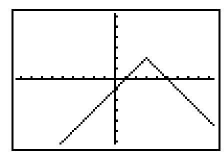
44. Line through (–1, 0) and (0, –3):
$m=\frac{-3-0}{0-(-1)}=\frac{-3}{1}=-3,$ so $y = -3x - 3$.
Line through (0, 3) and (2, –1):
$m=\frac{-1-3}{2-0}=\frac{-4}{2}=-2,$ so $y = -2x + 3$.
$f(x)=\begin{cases}-3x-3, & -1<x\le 0\\ -2x+3, & 0<x\le 2\end{cases}$

45. Line through (–1, 1) and (0, 0): $y = -x$
Line through (0, 1) and (1, 1): $y = 1$
Line through (1, 1) and (3, 0):
$m=\frac{0-1}{3-1}=\frac{-1}{2}=-\frac{1}{2},$ so
$y=-\frac{1}{2}(x-1)+1=-\frac{1}{2}x+\frac{3}{2}.$
$f(x)=\begin{cases}-x, & -1\le x<0\\ 1, & 0<x\le 1\\ -\frac{1}{2}x+\frac{3}{2}, & 1<x<3\end{cases}$

46. Line through (–2, –1) and (0, 0): $y=\frac{1}{2}x$
Line through (0, 2) and (1, 0): $y = -2x + 2$
Line through (1, –1) and (3, –1): $y = -1$
$f=\begin{cases}\frac{1}{2}x, & -2\le x\le 0\\ -2x+2, & 0<x\le 1\\ -1, & 1<x\le 3\end{cases}$

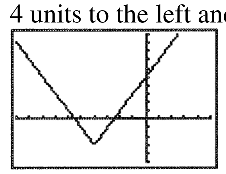
47. Line through $\left(\frac{T}{2}, 0\right)$ and $(T, 1)$:
$m=\frac{1-0}{T-\left(\frac{T}{2}\right)}=\frac{2}{T},$ so
$y=\frac{2}{T}\left(x-\frac{T}{2}\right)+0=\frac{2}{T}x-1.$
$f(x)=\begin{cases}0, & 0\le x\le \frac{T}{2}\\ \frac{2}{T}x-1, & \frac{T}{2}<x\le T\end{cases}$

48. $f(x)=\begin{cases}A, & 0\le x<\frac{T}{2}\\ -A, & \frac{T}{2}\le x<T\\ A, & T\le x<\frac{3T}{2}\\ -A & \frac{3T}{2}\le x\le 2T\end{cases}$

49. **(a)**

[–9.4, 9.4] by [–6.2, 6.2]
Note that $f(x) = -|x - 3| + 2$, so its graph is the graph of the absolute value function reflected across the x-axis and then shifted 3 units right and 2 units upward.

(b) $(-\infty, \infty)$

(c) $(-\infty, 2]$

50. **(a)** The graph of $f(x)$ is the graph of the absolute value function stretched vertically by a factor of 2 and then shifted 4 units to the left and 3 units downward.

[–10, 5] by [–5, 10]

(b) $(-\infty, \infty)$ or all real numbers

(c) $[-3, \infty)$

51. **(a)** $f(g(x)) = (x^2 - 3) + 5 = x^2 + 2$

(b) $g(f(x)) = (x+5)^2 - 3$
$= (x^2 + 10x + 25) - 3$
$= x^2 + 10x + 22$

(c) $f(g(0)) = 0^2 + 2 = 2$

(d) $g(f(0)) = 0^2 + 10 \cdot 0 + 22 = 22$

(e) $g(g(-2)) = [(-2)^2 - 3]^2 - 3 = 1^2 - 3 = -2$

(f) $f(f(x)) = (x + 5) + 5 = x + 10$

52. **(a)** $f(g(x)) = (x - 1) + 1 = x$

(b) $g(f(x)) = (x + 1) - 1 = x$

(c) $f(g(0)) = 0$

(d) $g(f(0)) = 0$

(e) $g(g(-2)) = (-2 - 1) - 1 = -3 - 1 = -4$

(f) $f(f(x)) = (x + 1) + 1 = x + 2$

53. **(a)** Since $(f \circ g)(x) = \sqrt{g(x) - 5} = \sqrt{x^2 - 5}$, $g(x) = x^2$.

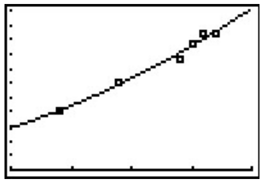
(b) Since $(f \circ g)(x) = 1 + \dfrac{1}{g(x)} = x$, we know that $\dfrac{1}{g(x)} = x - 1$, so $g(x) = \dfrac{1}{x-1}$.

(c) $(f \circ g)(x) = f\left(\dfrac{1}{x}\right) = x$, $f(x) = \dfrac{1}{x}$.

(d) Since $(f \circ g)(x) = f\left(\sqrt{x}\right) = |x|$, $f(x) = x^2$.
Completed table is shown. Note that the absolute value sign in part (d) is optional.

$g(x)$	$f(x)$	$(f \circ g)(x)$
x^2	$\sqrt{x-5}$	$\sqrt{x^2 - 5}$
$\frac{1}{x-1}$	$1+\frac{1}{x}$	$x, x \neq 1$
$\frac{1}{x}$	$\frac{1}{x}$	$x, x \neq 0$
$\sqrt{x}$	x^2	$\lvert x \rvert, x \geq 0$

54. **(a)** $y = 0.722x^2 - 2845.441x + 2{,}804{,}913.336$

(b) 

[1990, 2010] by [0, 1100]

(c) $y = 0.722(2012)^2 - 2845.441(2012) + 2{,}804{,}913.336$
$= 2646$ million or \$2.646 billion

(d) linear regression: $y = 41.869x - 83{,}088.89$
$y = 41.869(2012) - 83{,}088.89 = \$1152$ million in 2012

55. **(a)** Because the circumference of the original circle was 8π and a piece of length x was removed.

(b) $r = \dfrac{8\pi - x}{2\pi} = 4 - \dfrac{x}{2\pi}$

(c)
$$\begin{aligned} h &= \sqrt{16 - r^2} \\ &= \sqrt{16 - \left(4 - \frac{x}{2\pi}\right)^2} \\ &= \sqrt{16 - \left(16 - \frac{4x}{\pi} + \frac{x^2}{4\pi^2}\right)} \\ &= \sqrt{\frac{4x}{\pi} - \frac{x^2}{4\pi^2}} \\ &= \frac{\sqrt{16\pi x - x^2}}{2\pi} \end{aligned}$$

(d)
$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi\left(\frac{8\pi - x}{2\pi}\right)^2 \cdot \frac{\sqrt{16\pi x - x^2}}{2\pi} \\ &= \frac{(8\pi - x)^2\sqrt{16\pi x - x^2}}{24\pi^2} \end{aligned}$$

56. **(a)** Note that 2 mi = 10,560 ft, so there are $\sqrt{800^2 + x^2}$ feet of river cable at \$180 per foot and $(10{,}560 - x)$ feet of land cable at \$100 per foot. The cost is $C(x) = 180\sqrt{800^2 + x^2} + 100(10{,}560 - x)$.

(b) $C(0) = \$1{,}200{,}000$
$C(500) \approx \$1{,}175{,}812$
$C(1000) \approx \$1{,}186{,}512$
$C(1500) \approx \$1{,}212{,}000$
$C(2000) \approx \$1{,}243{,}732$
$C(2500) \approx \$1{,}278{,}479$
$C(3000) \approx \$1{,}314{,}870$
Values beyond this are all larger. It would appear that the least expensive location is less than 2000 ft from point P.

57. False: $x^4 + x^2 + x \neq (-x)^4 + (-x)^2 + (-x)$.

58. True: $(-x)^{-3} = -x^{-3}$

59. B: Since $9 - x^2 > 0$, the domain is $(-3, 3)$.

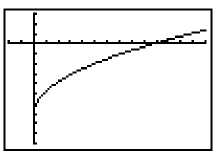
60. A: $y \neq 1$

61. D: $(f \circ g)(x) = 2(g(x)) - 1$
$= 2(x+3) - 1$
$= 2x + 5$
$(f \circ g) = (2) = 2(2) + 5 = 9$

62. C: $A(W) = LW$
$L = 2W$
$A(W) = 2W^2$

63. **(a)** Enter $y_1 = f(x) = x - 7$, $y_2 = g(x) = \sqrt{x}$,
$y_3 = (f \circ g)(x) = y_1(y_2(x))$, and
$y_4 = (g \circ f)(x) = y_2(y_1(x))$

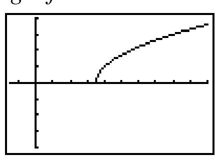
$f \circ g$:



[–10, 70] by [–10, 3]

Domain: $[0, \infty)$
Range: $[-7, \infty)$

$g \circ f$:



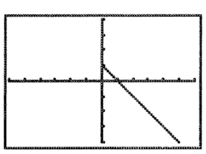
[–3, 20] by [–4, 4]

Domain: $[7, \infty)$
Range: $[0, \infty)$

(b) $(f \circ g)(x) = \sqrt{x} - 7$
$(g \circ f)(x) = \sqrt{x - 7}$

64. **(a)** Enter $y_1 = f(x) = 1 - x^2$,
$y_2 = g(x) = \sqrt{x}$,
$y_3 = (f \circ g)(x) = y_1(y_2(x))$, and
$y_4 = (g \circ f)(x) = y_2(y_1(x))$.

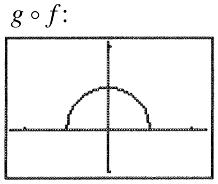
$f \circ g$:



[−6, 6] by [−4, 4]

Domain: $[0, \infty)$
Range: $(-\infty, 1]$

$g \circ f$:



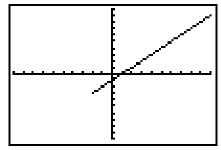
[−2.35, 2.35] by [−1, 2.1]

Domain: $[-1, 1]$
Range: $[0, 1]$

(b) $(f \circ g)(x) = 1 - \left(\sqrt{x}\right)^2 = 1 - x,\ x \geq 0$
$(g \circ f)(x) = \sqrt{1 - x^2}$

65. **(a)** Enter $y_1 = f(x) = x^2 - 3$,
$y_2 = g(x) = \sqrt{x+2}$,
$y_3 = (f \circ g)(x) = y_1(y_2(x))$, and
$y_4 = (g \circ f)(x) = y_2(y_1(x))$.

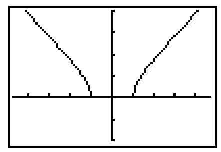
$f \circ g$:



[–10, 10] by [–10, 10]

Domain: $[-2, \infty)$
Range: $[-3, \infty)$

$g \circ f$:



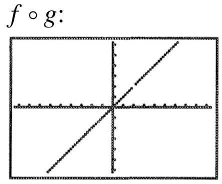
[–4.7, 4.7] by [–2, 4]

Domain: $(-\infty, -1] \cup [1, \infty)$
Range: $[0, \infty)$

(b) $(f \circ g)(x) = \left(\sqrt{x+2}\right)^2 - 3$
$= (x+2) - 3,\ x \geq -2$
$= x - 1,\ x \geq -2$
$(g \circ f)(x) = \sqrt{(x^2 - 3) + 2} = \sqrt{x^2 - 1}$

66. **(a)** Enter $y_1(x) = f(x) = \dfrac{2x-1}{x+3}$, $y_2 = \dfrac{3x+1}{2-x}$.
$y_3 = (f \circ g)(x) = y_1(y_2(x))$, and
$y_4 = (g \circ f)(x) = y_2(y_1(x))$.
Use a "decimal window" such as the one shown.

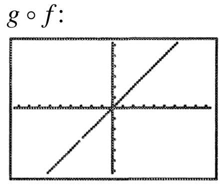
$f \circ g$:



[−9.4, 9.4] by [−6.2, 6.2]

Domain: $(-\infty, 2) \cup (2, \infty)$
Range: $(-\infty, 2) \cup (2, \infty)$

$g \circ f$:



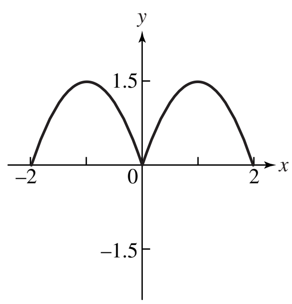
[−9.4, 9.4] by [−6.2, 6.2]

Domain: $(-\infty, -3) \cup (-3, \infty)$
Range: $(-\infty, -3) \cup (-3, \infty)$

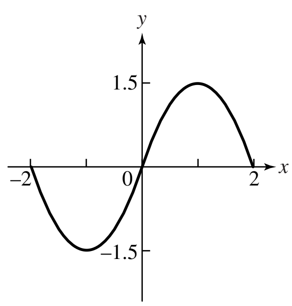
(b) $$(f \circ g)(x) = \frac{2\left(\frac{3x+1}{2-x}\right)-1}{\frac{3x+1}{2-x}+3}$$
$$= \frac{2(3x+1)-(2-x)}{(3x+1)+3(2-x)},\ x \neq 2$$
$$= \frac{7x}{7},\ x \neq 2$$
$$= x,\ x \neq 2$$

$$(g \circ f)(x) = \frac{3\left(\frac{2x-1}{x+3}\right)+1}{2-\frac{2x-1}{x+3}}$$
$$= \frac{3(2x-1)+(x+3)}{2(x+3)-(2x-1)},\ x \neq -3$$
$$= \frac{7x}{7},\ x \neq -3$$
$$= x,\ x \neq -3$$

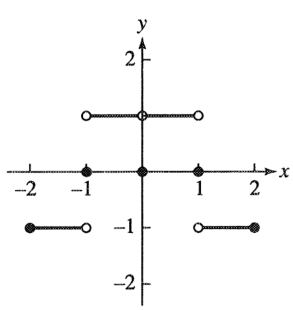
67. (a)



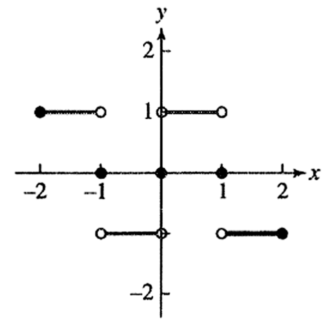
(b)



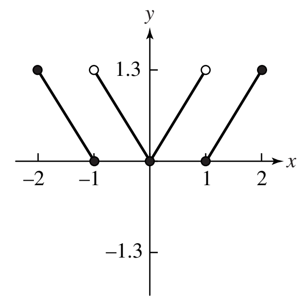
68. (a)



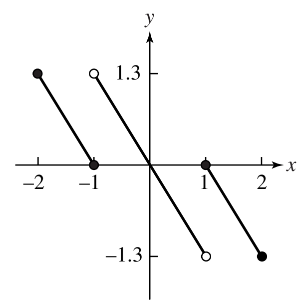
(b)



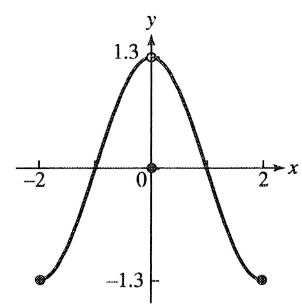
69. (a)



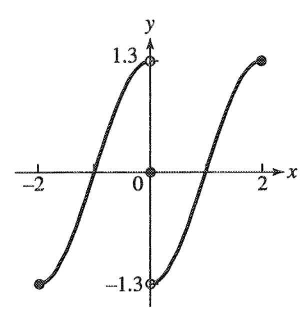
(b)



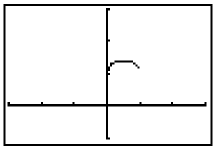
70. (a)



(b)



71. **(a)**



[–3, 3] by [–1, 3]

(b) Domain of y_1: $[0, \infty)$

Domain of y_2: $(-\infty, 1]$

Domain of y_3: $[0, 1]$

(c) The functions $y_1 - y_2$, $y_2 - y_1$, and $y_1 \cdot y_2$ all have domain [0, 1], the same as the domain of $y_1 + y_2$ found in part (b).

Domain of $\dfrac{y_1}{y_2}$: $[0, 1)$

Domain of $\dfrac{y_2}{y_1}$: $(0, 1]$

(d) The domain of a sum, difference, or product of two functions is the intersection of their domains. The domain of a quotient of two functions is the intersection of their domains with any zeros of the denominator removed.

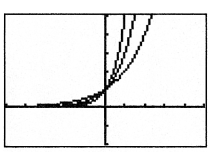
72. **(a)** Yes, since
$$\begin{aligned}(f \cdot g)(-x) &= f(-x) \cdot g(-x)\\ &= f(x) \cdot g(x)\\ &= (f \cdot g)(x),\end{aligned}$$
function $(f \cdot g)(x)$ will also be even.

(b) The product will be even, since
$$\begin{aligned}(f \cdot g)(-x) &= f(-x) \cdot g(-x)\\ &= (-f(x)) \cdot (-g(x))\\ &= f(x) \cdot g(x)\\ &= (f \cdot g)(x).\end{aligned}$$

Section 1.3 Exponential functions (pp. 22–28)

Exploration 1 Exponential Functions

1.



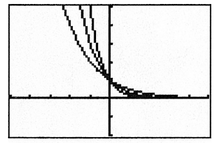
[−5, 5] by [−2, 5]

2. $x > 0$

3. $x < 0$

4. $x = 0$

5.



[−5, 5] by [−2, 5]

6. $2^{-x} < 3^{-x} < 5^{-x}$ for $x < 0$; $2^{-x} > 3^{-x} > 5^{-x}$ for $x > 0$; $2^{-x} = 3^{-x} = 5^{-x}$ for $x = 0$.

Quick Review 1.3

1. Using a calculator, $5^{2/3} \approx 2.924$.

2. Using a calculator, $3^{\sqrt{2}} \approx 4.729$.

3. Using a calculator, $3^{-1.5} \approx 0.192$.

4.
$$\begin{aligned}x^3 &= 17\\ x &= \sqrt[3]{17}\\ x &\approx 2.5713\end{aligned}$$

5.
$$\begin{aligned}x^5 &= 24\\ x &= \sqrt[5]{24}\\ x &\approx 1.8882\end{aligned}$$

6.
$$\begin{aligned}x^{10} &= 1.4567\\ x &= \pm\sqrt[10]{1.4567}\\ x &\approx \pm 1.0383\end{aligned}$$

7. $\$500(1.0475)^5 \approx \$630.58$

8. $\$1000(1.063)^3 \approx \$1201.16$

9.
$$\begin{aligned}\frac{(x^{-3}y^2)^2}{(x^4y^3)^3} &= \frac{x^{-6}y^4}{x^{12}y^9}\\ &= x^{-6-12}y^{4-9}\\ &= x^{-18}y^{-5}\\ &= \frac{1}{x^{18}y^5}\end{aligned}$$

10. $\left(\dfrac{a^3b^{-2}}{c^4}\right)^2\left(\dfrac{a^4c^{-2}}{b^3}\right)^{-1} = \dfrac{a^6b^{-4}}{c^8}\cdot\dfrac{b^3}{a^4c^{-2}}$

$= \dfrac{a^6}{b^4c^8}\cdot\dfrac{b^3c^2}{a^4}$

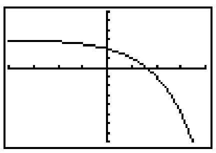
$= a^{6-4}b^{-4+3}c^{-8+2}$

$= a^2b^{-1}c^{-6}$

$= \dfrac{a^2}{bc^6}$

Section 1.3 Exercises

1.

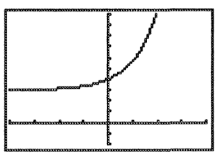


[–4, 4] by [–8, 6]

Domain: $(-\infty, \infty)$

Range: $(-\infty, 3)$

2.

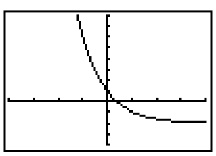


[−4, 4] by [−2, 10]

Domain: $(-\infty, \infty)$

Range: $(3, \infty)$

3.

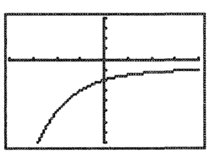


[–4, 4] by [–4, 8]

Domain: $(-\infty, \infty)$

Range: $(-2, \infty)$

4.



[−4, 4] by [−8, 4]

Domain: $(-\infty, \infty)$

Range: $(-\infty, -1)$

5. $9^{2x} = (3^2)^{2x} = 3^{4x}$

6. $16^{3x} = (2^4)^{3x} = 2^{12x}$

7. $\left(\dfrac{1}{8}\right)^{2x} = (2^{-3})^{2x} = 2^{-6x}$

8. $\left(\dfrac{1}{27}\right)^{x} = (3^{-3})^{x} = 3^{-3x}$

9. zero: ≈ 2.322

10. zero: ≈ 1.386

11. zero: ≈ -0.631

12. zero: ≈ 1.585

13. The graph of $y = 2^x$ is increasing from left to right and has the negative x-axis as an asymptote. (a)

14. The graph of $y = 3^{-x}$ or, equivalently, $y = \left(\dfrac{1}{3}\right)^x$, is decreasing from left to right and has the positive x-axis as an asymptote. (d)

15. The graph of $y = -3^{-x}$ is the reflection about the x-axis of the graph in Exercise 14. (e)

16. The graph of $y = -0.5^{-x}$ or, equivalently, $y = -2^x$, is the reflection about the x-axis of the graph in Exercise 13. (c)

17. The graph of $y = 2^{-x} - 2$ is decreasing from left to right and has the line $y = -2$ as an asymptote. (b)

18. The graph of $y = 1.5^x - 2$ is increasing from left to right and has the line $y = -2$ as an asymptote. (f)

19. **(a)** $\dfrac{2238}{2168} \approx 1.032$

$\dfrac{2330}{2238} \approx 1.041$

$\dfrac{2409}{2330} \approx 1.034$

$\dfrac{2492}{2409} \approx 1.034$

$\dfrac{2565}{2492} \approx 1.029$

(b) One possibility is $2168(1.034)^n$

(c) $2168(1.034)^{13} = 3348$ thousand or 3,348,000

20. **(a)** $\frac{7371}{7282} \approx 1.012$

$\frac{7464}{7371} \approx 1.013$

$\frac{7558}{7464} \approx 1.013$

$\frac{7640}{7558} \approx 1.011$

$\frac{7712}{7640} \approx 1.009$

(b) One possibility is $7282(1.012)^n$.

(c) $7282(1.012)^{10} = 8205$ thousand or 8,205,000

21. Let t be the number of years. Solving $500,000(1.0375)^t = 1,000,000$ graphically, we find that $t \approx 18.828$. The population will reach 1 million in about 19 years.

22. **(a)** The population is given by $P(t) = 6250(1.0275)^t$, where t is the number of years after 1890.
Population in 1915: $P(25) \approx 12,315$
Population in 1940: $P(50) \approx 24,265$

(b) Solving $P(t) = 50,000$ graphically, we find that $t \approx 76.651$. The population reached 50,000 about 77 years after 1890, in 1967.

23. **(a)** $A(t) = 6.6\left(\frac{1}{2}\right)^{t/14}$

(b) Solving $A(t) = 1$ graphically, we find that $t \approx 38.1145$. There will be 1 gram remaining after about 38.1145 days.

24. Let t be the number of years. Solving $2300(1.06)^t = 4150$ graphically, we find that $t \approx 10.129$. It will take about 10.129 years. (If the interest is not credited to the account until the end of each year, it will take 11 years.)

25. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A(1.0625)^t = 2A$, which is equivalent to $1.0625^t = 2$. Solving graphically, we find that $t \approx 11.433$. It will take about 11.433 years. (If the interest is credited at the end of each year, it will take 12 years.)

26. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A\left(1+\frac{0.0625}{12}\right)^{12t} = 2A$, which is equivalent to $\left(1+\frac{0.0625}{12}\right)^{12t} = 2$. Solving graphically, we find that $t \approx 11.119$. It will take about 11.119 years. (If the interest is credited at the end of each month, it will take 11 years, 2 months.)

27. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $Ae^{0.0625t} = 2A$, which is equivalent to $e^{0.0625t} = 2$. Solving graphically, we find that $t \approx 11.090$. It will take about 11.090 years.

28. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A(1.0575)^t = 3A$, which is equivalent to $1.0575^t = 3$. Solving graphically, we find that $t \approx 19.650$. It will take about 19.650 years. (If the interest is credited at the end of each year, it will take 20 years.)

29. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A\left(1+\frac{0.0575}{365}\right)^{365t} = 3A$, which is equivalent to $\left(1+\frac{0.0575}{365}\right)^{365t} = 3$. Solving graphically, we find that $t \approx 19.108$. It will take about 19.108 years.

30. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $Ae^{0.0575t} = 3A$, which is equivalent to $e^{0.0575t} = 3$. Solving graphically, we find that $t \approx 19.106$. It will take about 19.106 years.

31. After t hours, the population is $P(t) = 2^{t/0.5}$ or, equivalently, $P(t) = 2^{2t}$. After 24 hours, the population is $P(24) = 2^{48} \approx 2.815 \times 10^{14}$ bacteria.

32. **(a)** Each year, the number of cases is 100% – 20% = 80% of the previous year's number of cases. After t years, the number of cases will be $C(t) = 10{,}000(0.8)^t$. Solving $C(t) = 1000$ graphically, we find that $t \approx 10.319$. It will take about 10.319 years.

(b) Solving $C(t) = 1$ graphically, we find that $t \approx 41.275$. It will take about 41.275 years.

33.

x	y	Δy
1	–1	
		2
2	1	
		2
3	3	
		2
4	5	

34.

x	y	Δy
1	1	
		–3
2	–2	
		–3
3	–5	
		–3
4	–8	

35.

x	y	Δy
1	1	
		3
2	4	
		5
3	9	
		7
4	16	

36.

x	y	ratio
1	8.155	
		2.718
2	22.167	
		2.718
3	60.257	
		2.718
4	163.794	

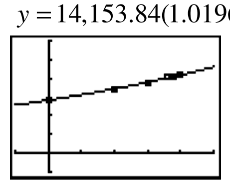
37. Since $\Delta x = 1$, the corresponding value of Δy is equal to the slope of the line. If the changes in x are constant for a linear function, then the corresponding changes in y are constant as well.

38. **(a)** When $t = 0$, $B = 100e^0 = 100$. There were 100 bacteria present initially.

(b) When $t = 6$, $B = 100e^{0.639(6)} \approx 6394.351$. After 6 hours, there are about 6394 bacteria.

(c) Solving $100e^{0.639t} = 200$ graphically, we find that $t \approx 1.000$. The population will be 200 after about 1 hour. Since the population doubles (from 100 to 200) in about 1 hour, the doubling time is about 1 hour.

39. **(a)** $y = 14{,}153.84(1.01963)^x$

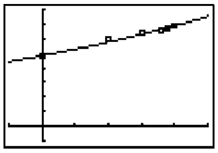


[−5, 25] by [−5000, 30000]

(b) Estimate:
$14{,}153.84(1.01963)^{23} = 22{,}134$ thousand or 22,134,000.
22,134,000 – 22,119,000 = 15,000
The estimate exceeds the actual by 15,000.

(c) $0.01963 \approx 0.020$ or 2%

40. **(a)** $y = 24{,}121.49(1.0178)^x$



[−5, 25] by [−5000, 40000]

(b) Estimate:

$24{,}121.49(1.0178)^{23} = 36{,}194$ thousand or 36,194,000.

36,194,000 − 35,484,000 = 710,000

The estimate exceeds the actual by 710,000.

(c) $0.0178 \approx 0.018$ or 1.8%

41. False; $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$; it is positive.

42. True; $4^3 = (2^2)^3 = 2^6$

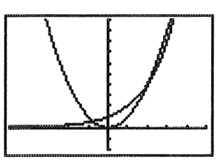
43. D: Let t be the number of years. We wish to solve $100(1.045)^t = 200$, which is equivalent to $1.045^t = 2$. Solving graphically, we find $t \approx 15.747$ years.

44. A

45. B

46.
$$0 = 4 - e^x$$
$$e^x = 4$$
$$x = \ln 4$$
$$x = 1.386$$

47. **(a)**



[−5, 5] by [−2, 10]

In this window, it appears they cross twice, although a third crossing off-screen appears likely.

(b)

x	change in $y1$	change in $y2$
1		
	3	2
2		
	5	4
3		
	7	8
4		

It happens by the time $x = 4$.

(c) Solving graphically, $x \approx -0.7667$, $x = 2$, $x = 4$.

(d) The solution set is approximately $(-0.7667, 2) \cup (4, \infty)$.

48. Since $f(1) = 4.5$ we have $ka = 4.5$, and since $f(-1) = 0.5$ we have $ka^{-1} = 0.5$.
$$\frac{ka}{ka^{-1}} = \frac{4.5}{0.5}$$
Dividing, we have
$$a^2 = 9$$
$$a = \pm 3$$
Since $f(x) = k \cdot a^x$ is an exponential function, we require $a > 0$, so $a = 3$. Then $ka = 4.5$ gives $3k = 4.5$, so $k = 1.5$. The values are $a = 3$ and $k = 1.5$.

49. Since $f(1) = 1.5$ we have $ka = 1.5$, and since $f(-1) = 6$ we have $ka^{-1} = 6$. Dividing, we have
$$\frac{ka}{ka^{-1}} = \frac{1.5}{6}$$
$$a^2 = 0.25$$
$$a = \pm 0.5$$
Since $f(x) = k \cdot a^x$ is an exponential function, we require $a > 0$, so $a = 0.5$. Then $ka = 1.5$ gives $0.5k = 1.5$, so $k = 3$. The values are $a = 0.5$ and $k = 3$.

Quick Quiz (Sections 1.1–1.3)

1. C, $m = -2$
$$y = -2(x-3) + (-1)$$
$$y = -2x + 5$$

2. D, $g(2) = 2(2) - 1 = 4 - 1 = 3$

$f(3) = (3)^2 + 1 = 9 + 1 = 10$

3. E, $A(t) = 5\left(\frac{1}{2}\right)^{t/8}$

We need to solve $A(t) = 1$. Solving graphically, we find $t \approx 18.6$ years.

4. **(a)** $(-\infty, \infty)$

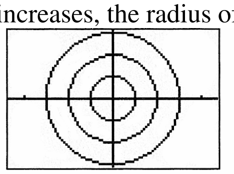
(b) $(-2, \infty)$

(c)
$$\begin{aligned} 0 &= e^{-x} - 2 \\ e^{-x} &= 2 \\ -x &= \ln 2 \\ x &= -0.693 \end{aligned}$$

Section 1.4 Parametric Equations (pp. 29–35)

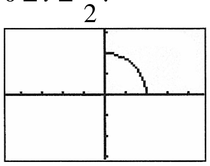
Exploration 1 Parametrizing Circles

1. Each is a circle with radius $|a|$. As $|a|$ increases, the radius of the circle increases.



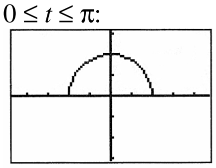
[−4.7, 4.7] by [−3.1, 3.1]

2. $0 \le t \le \frac{\pi}{2}$:



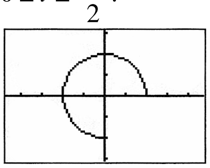
[−4.7, 4.7] by [−3.1, 3.1]

$0 \le t \le \pi$:



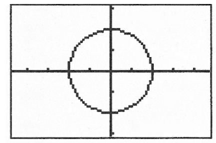
[−4.7, 4.7] by [−3.1, 3.1]

$0 \le t \le \frac{3\pi}{2}$:



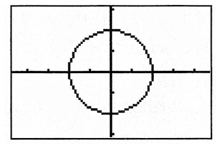
[−4.7, 4.7] by [−3.1, 3.1]

$2\pi \le t \le 4\pi$:



[−4.7, 4.7] by [−3.1, 3.1]

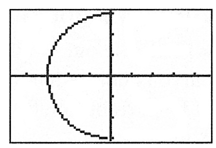
$0 \le t \le 4\pi$



[−4.7, 4.7] by [−3.1, 3.1]

Let d be the length of the parametric interval. If $d < 2\pi$, you get $\frac{d}{2\pi}$ of a complete circle. If $d = 2\pi$, you get the complete circle. If $d > 2\pi$, you get the complete circle but portions of the circle will be traced out more than once. For example, if $d = 4\pi$ the entire circle is traced twice.

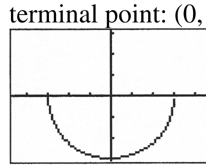
3.



$\frac{\pi}{2} \le t \le \frac{3\pi}{2}$

initial point: (0, 3)

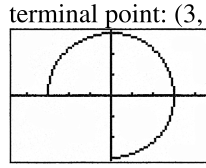
terminal point: (0, –3)



$\pi \le t \le 2\pi$

initial point: (–3, 0)

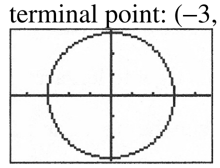
terminal point: (3, 0)



$\frac{3\pi}{2} \le t \le 3\pi$

initial point: (0, –3)

terminal point: (–3, 0)



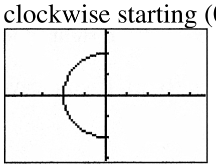
$\pi \le t \le 5\pi$

initial point: (–3, 0)

terminal point: (–3, 0)

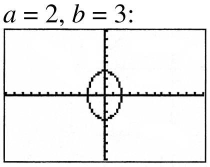
4. For $0 \le t \le 2\pi$, the complete circle is traced once clockwise beginning and ending at (2, 0). For $\pi \le t \le 3\pi$, the complete circle is traced once clockwise beginning and ending at (−2, 0).

For $\frac{\pi}{2} \le t \le \frac{3\pi}{2}$ the half circle below is traced clockwise starting (0, −2) and ending at (0, 2).



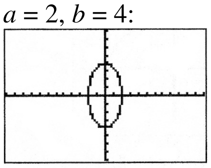
Exploration 2 Parametrizing Ellipses

1. $a = 2, b = 3$:



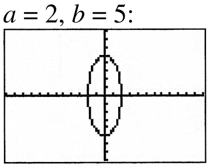
[−12, 12] by [−8, 8]

$a = 2, b = 4$:



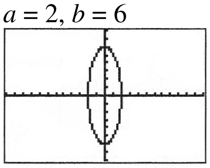
[−12, 12] by [−8, 8]

$a = 2, b = 5$:



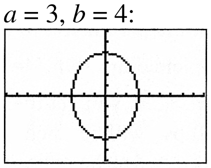
[−12, 12] by [−8, 8]

$a = 2, b = 6$



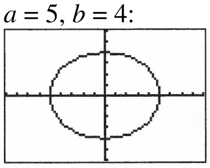
[−12, 12] by [−8, 8]

2. $a = 3, b = 4$:



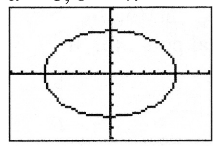
[−9, 9] by [−6, 6]

$a = 5, b = 4$:



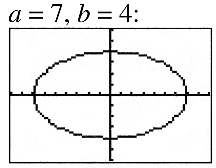
[−9, 9] by [−6, 6]

$a = 6, b = 4$:



[−9, 9] by [−6, 6]

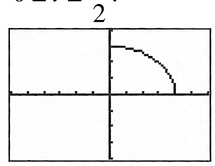
$a = 7, b = 4$:



[−9, 9] by [−6, 6]

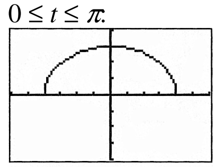
3. If $|a| > |b|$, then the major axis is on the x-axis and the minor on the y-axis. If $|a| < |b|$, then the major axis is on the y-axis and the minor on the x-axis.

4. $0 \le t \le \frac{\pi}{2}$:



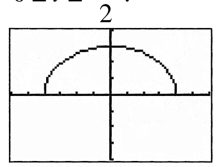
[−6, 6] by [−4, 4]

$0 \le t \le \pi$:



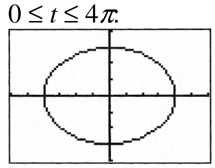
[−6, 6] by [−4, 4]

$0 \le t \le \frac{3\pi}{2}$:



[−6, 6] by [−4, 4]

$0 \le t \le 4\pi$:



[−6, 6] by [−4, 4]

Let d be the length of the parametric interval. If $d < 2\pi$, you get $\frac{d}{2\pi}$ of a complete ellipse. If $d = 2\pi$, you get the complete ellipse. If $d > 2\pi$, you get the complete ellipse but portions of the ellipse will be traced out more than once. For example, if $d = 4\pi$ the entire ellipse is traced twice.